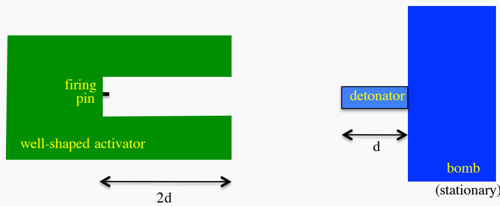
BOMB PARADOX

A stationary bomb is attached to a nub-shaped detonator that will blow the bomb when depressed. The length of the nub is "d" units. A firing pin needed to ignite the bomb is located at the bottom of a well-shaped activator. The well's depth is "2d." Both pieces, WHILE STATIONARY, are shown to the right.

At some point, the well-shaped activator is accelerated to a velocity "v." As presented, will the bomb go off?



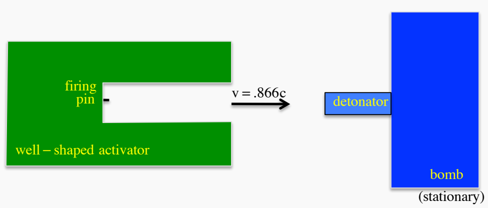
You really can't answer this unless you know more, so let's begin by assuming the well's velocity is .866c (note that the sketch is ***not to scale***—this problem will be remedied on the next slide). With that information, we can write the relative velocity as:

$$\beta = .866 = \frac{\sqrt{3}}{2}$$

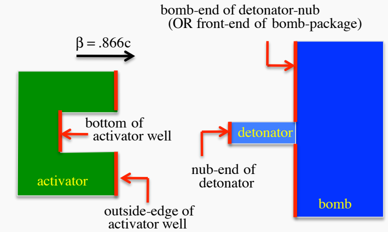
and the relativistic factor as:

$$\gamma = \frac{1}{\left[1-\left(\frac{v}{c}\right)^2\right]^{1/2}} = \frac{1}{\left[1-\left(\frac{.866c}{c}\right)^2\right]^{1/2}} = \frac{1}{\left[1-.75\right]^{1/2}}$$

$$\gamma = 2$$

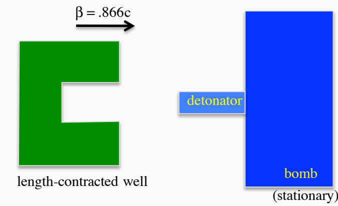


As a matter of convenience, the sketch to the right identifies each part of the system and how each will be termed. (Notice that with the activator's length contraction, the sketch is now to scale!)

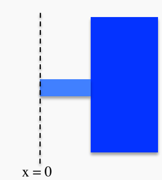


We will start by looking at the problem from the perspective of the bomb being stationary. That means the moving activator will length contract by a factor of $1/\gamma = .5$ leaving it with a well-depth of "d."

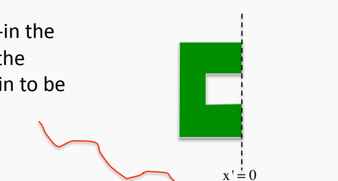
Although the scaled sketch suggests that the *nub-end of the detonator* will hit the *bottom of the activator well* (detonating the bomb) as the *bomb package* comes in contact with the *outside edge of the activator* (this would otherwise stop the incursion, or so one might think), a space-time diagram is really the best way to see what is happening in this problem. That is what we are about to look at.



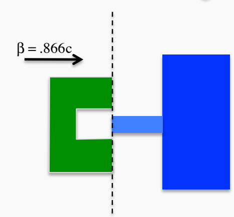
For our purposes, the unprimed coordinate axes (*x* and *ct*) will be associated with the bomb's frame of reference (again, we are starting out assuming the bomb is not what is moving). We need to define an origin. In this case, the ideal position for x = 0 is the *nub-end of the detonator* (see sketch).



We also need to define an origin--an x' = 0 point--in the activator's frame of reference (this will be called the prime frame of reference). We will take that origin to be located at the outside edge of the well.



We additionally need to decide when to "start the clock." Let's assume that when the *outside-edge of the activator-well* just passes the *nub-end of the detonator*, both clocks start (that is, ct = 0 and ct' = 0 at that point).



sketch of system at ct = 0 and ct' = 0

outside-edge of activator well just passing *nub-end of detonator* at ct = 0 and ct' = 0.

The only other two bits of amusement we need are more like reminders than anything else.

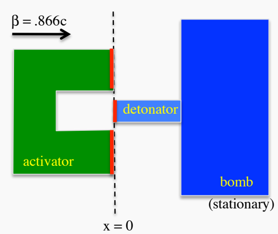
First, a world-line tracks the motion of a POINT ON AN OBJECT on a space-time diagram. It allows you to identify where the point is (i.e., its position coordinate “x”) at a given instant in time (i.e., its time coordinate “ct”).

That means that even though a point on an object may be physically stationary with its “x” coordinate not changing, that point will still be moving in time. In other words, a point’s world-line will always, at a minimum, show motion along the “ct” axis.

Second, an event can simply be a point of interest on a world-line, though it is more commonly associated with a point where two world-lines cross. For instance, if the world-line of the *bottom of the activator* (where the firing pin is) were to cross the world-line of the *nub-end of the detonator*, the firing pin and detonator would be in the same place at the same time . . . which is to say the bomb would blow. That event would occur when their world-lines crossed.

There are three points of interest (events) to examine for this problem. They are:

1.) event 1: The *outside edge of the activator-well* reaches the *nub-end of detonator*; this happens in the unprimed frame at x = 0, ct = 0.

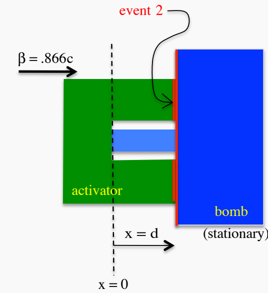


event 1
(the nub-end of detonator reaches outside edge of activator-well)

2.) event 2: The *outside-edge of the activator* aligns with the *bomb-end of the detonator-nub* (it also comes in contact with the *front edge of the bomb*); in bomb’s frame, this happens at x = d, ct = d/β.

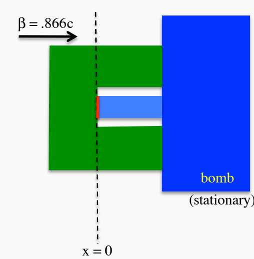
(Minor point: how did we get this time?)
The activator traveling with velocity "$v = \beta c$" moves a distance “d” in time “t.” That means:

$$v = d/t$$
$$\Rightarrow \quad \beta c = d/t$$
$$\Rightarrow \quad ct = d/\beta$$



event 2
(the outside edge of well reaches the bomb and aligns with the bomb-end of the nub)

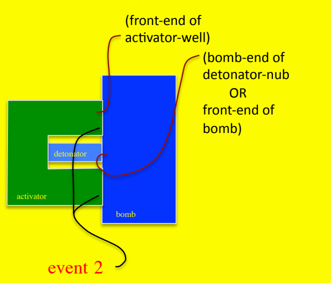
3.) event 3: *The bottom of activator-well* reaches the *nub-end of the detonator*; in bomb’s frame, this happens at x = 0 at time ct = d/β .



event 3
(the nub-end of detonator reaches bottom of well)

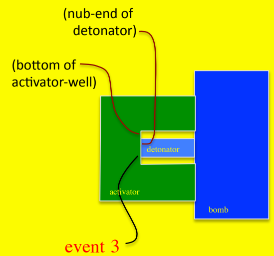
Comment: Notice that the sketch for event 3 looks just like the sketch for event 2. How so? Again, events are defined as points of interest along a world-line. If two separate events happen *simultaneously* in a given frame of reference (like the bomb's frame) but with different coordinates, the sketch that depicts each situation will look the same.

Event 2 is the point where the world-line of the *activator's front* crosses the world-line of the *bomb-end of the detonator-nub*.



(front-end of activator-well strikes bomb-end of detonator-nub—i.e., front-end of bomb)

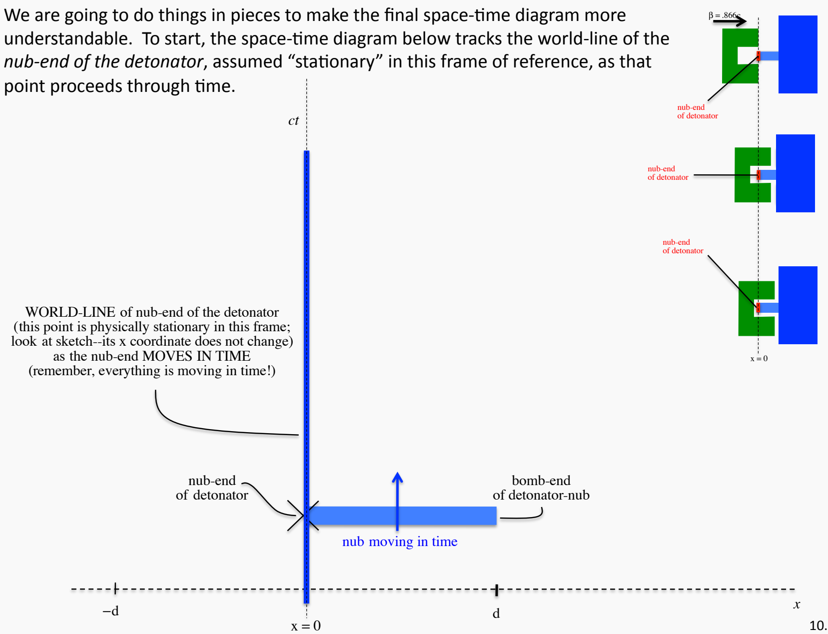
Event 3 is the point where the world-line of the *nub-end of the detonator* crosses the world-line of the *bottom of the activator-well* (i.e., where the firing pin is).



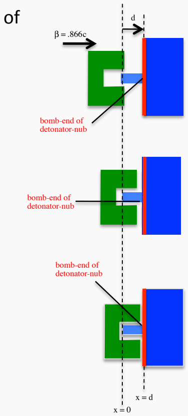
(bottom of activator-well strikes nub-end of detonator)

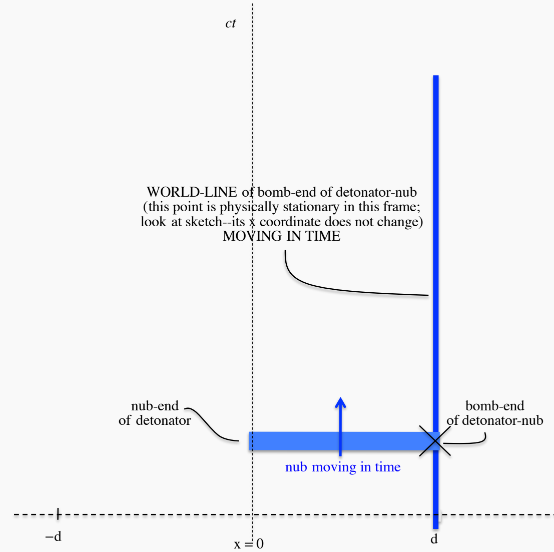
The two events happen simultaneously in the bomb's frame of reference, so the drawings that picture these two occurrences look the same.

We are going to do things in pieces to make the final space-time diagram more understandable. To start, the space-time diagram below tracks the world-line of the *nub-end of the detonator*, assumed "stationary" in this frame of reference, as that point proceeds through time.

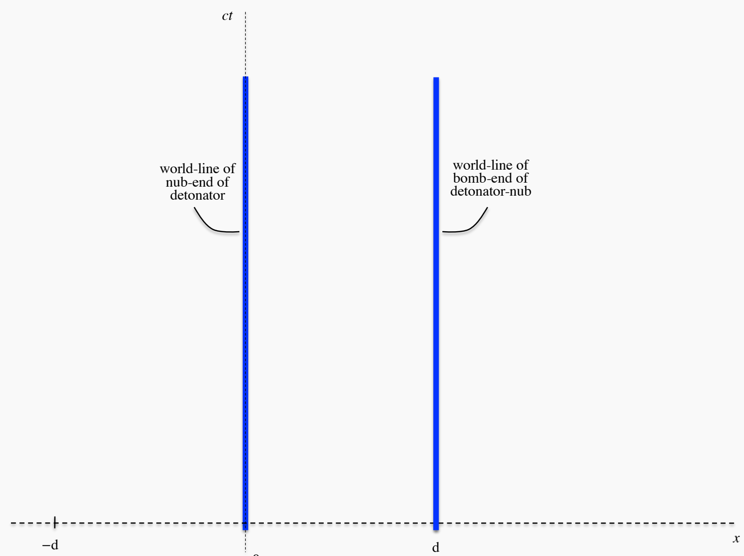


The space-time diagram below tracks the world-line of the *bomb-end of the detonator-nub* (aka: the *front of the bomb*) assumed "stationary" in this frame of reference, as that point proceeds through time.





The space-time diagram below shows the world-lines of the *nub-end of the detonator* and the *bomb-end of the detonator* (this is the way these lines would normally be presented).

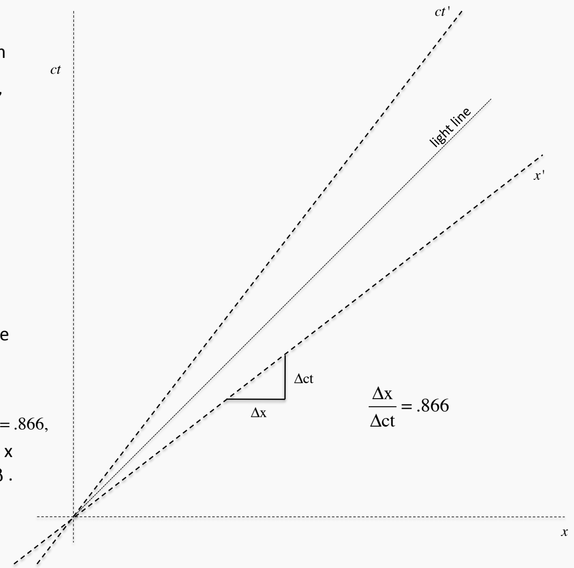


We would like to do a similarly thoughtful presentation of two world-lines associated with the *front-end of the activator-well* and *the bottom of the activator-well*. Before we do, we need to review a bit.

1.) Shown are the unprimed axes (x and ct) associated with the "stationary" reference frame and the primed axes (x' and ct') associated with the "moving" reference frame.

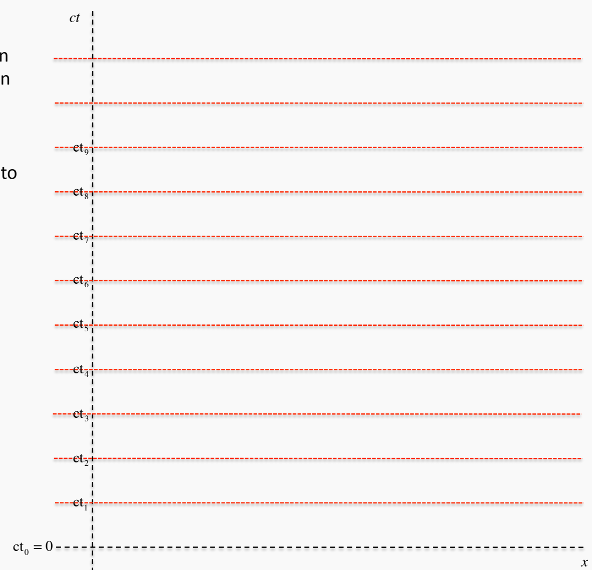
2.) The moving axes have world-lines that are not at right angles with one another and appear squashed as viewed from the unprimed frame. This is because those axes are moving relative to the "fixed" frame.

3.) The relative velocity between the two frames is $\beta = .866$, where the angle between the x and x' axes is determined by β .



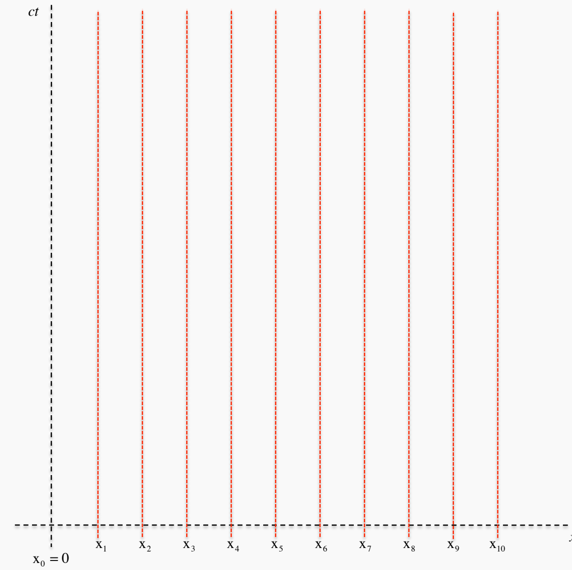
More review:

5.) A line of simultaneity (a line that includes all points in space associated with a given time) in the unprimed coordinate system runs parallel to the x-axis. A number of these are shown to the right.



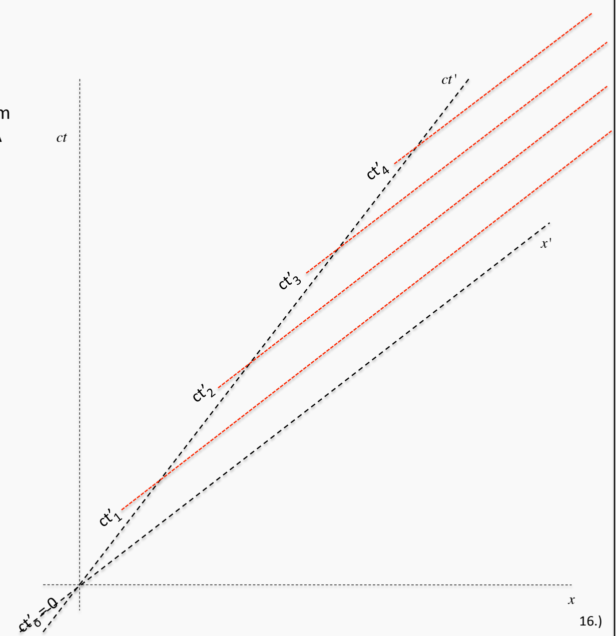
More review:

6.) A line of constant position (a line that includes all points in time associated with a given point in space) in the unprimed coordinate system run parallel to the ct-axis. A number of these lines are shown to the right.



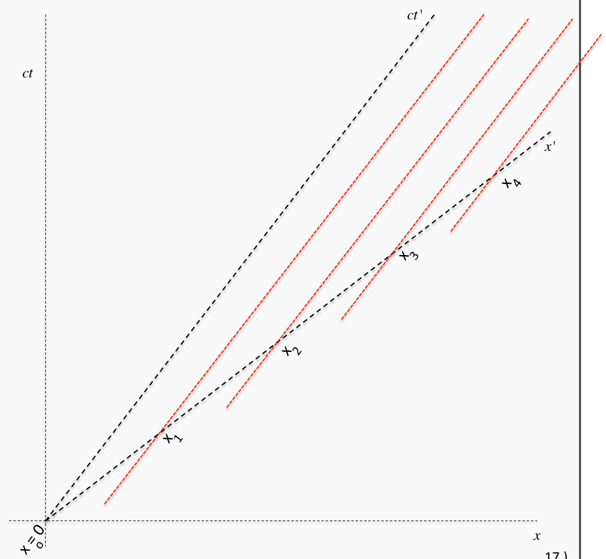
Still more review:

7.) A line of simultaneity in the primed coordinate system runs parallel to the x'-axis. A number of these lines are shown to the right.



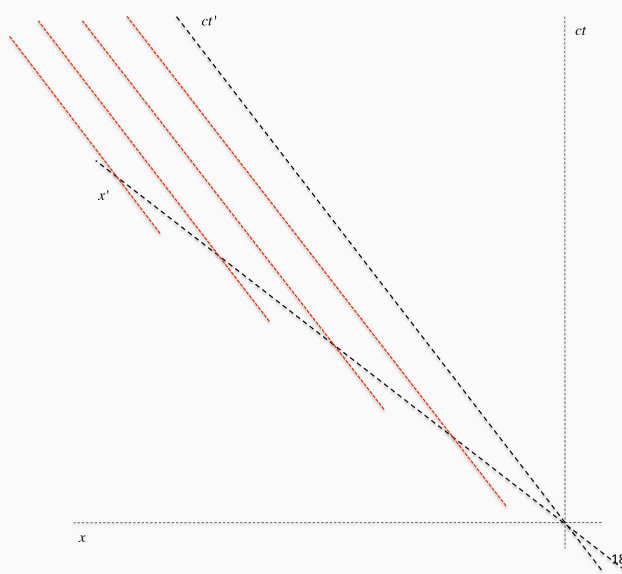
Still more review:

8.) A line of constant position in the primed coordinate system runs parallel to the ct'-axis. A number of these lines are shown to the right.



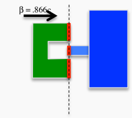
17.)

Note that if the relative motion was directed toward the left instead of toward the right, as would be the case if we were assuming it was the bomb that was moving toward the activator instead of vice versa, the diagram that included lines of simultaneity would have looked like:

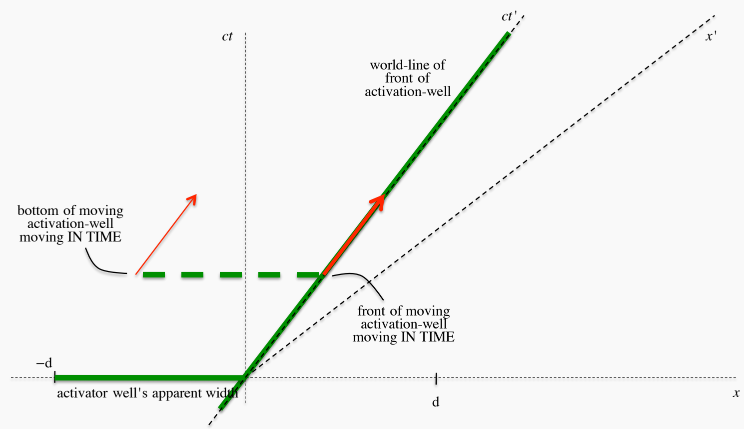


18.)

So what would the world-line of *the front of the activator-well* look like?

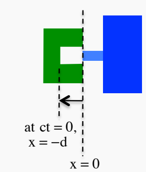


Well, as that point would be stationary in its own frame of reference (the primed frame), its x' coordinate would not change and, as a consequence, its world-line path would parallel the ct' axis. And as it is supposed to be at the origin of both the primed and unprimed coordinate systems at ct = 0 and ct' = 0 (we defined the axes that way), it looks as though its path will follow along the ct'= 0 line.

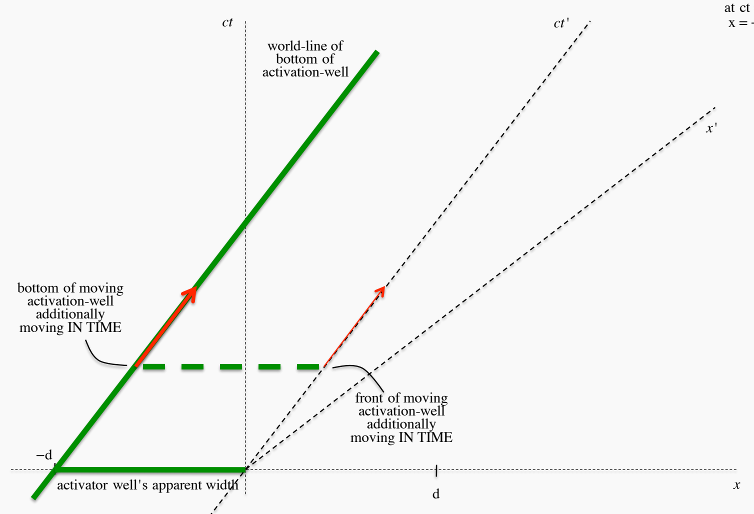


19.)

What would the world-line of *the bottom of the activator-well* look like?

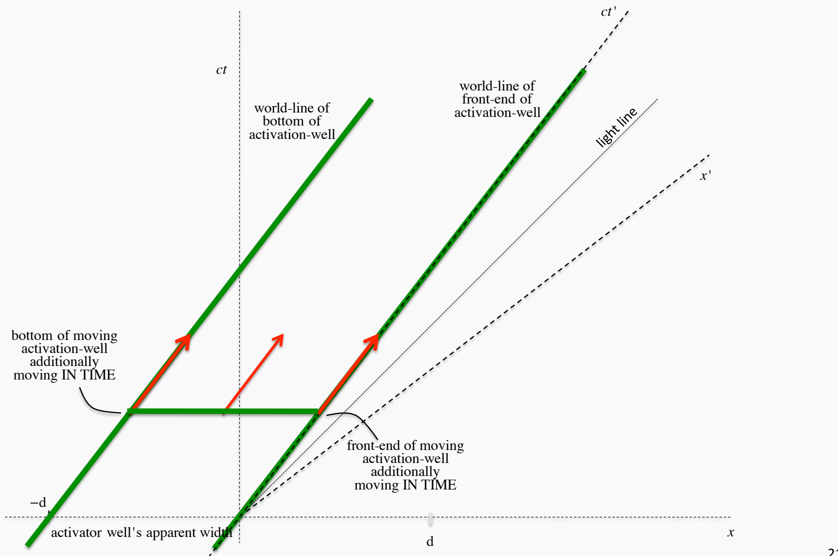


First, its world-line will parallel the world-line of the activator's front-end, which we already know. Also, at ct = 0, that point will be at x = -d. Putting this all together yields the line shown.

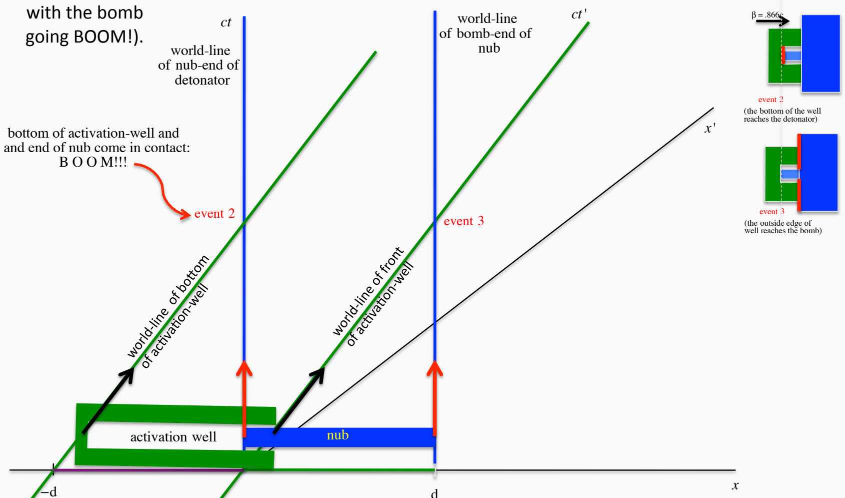


20.)

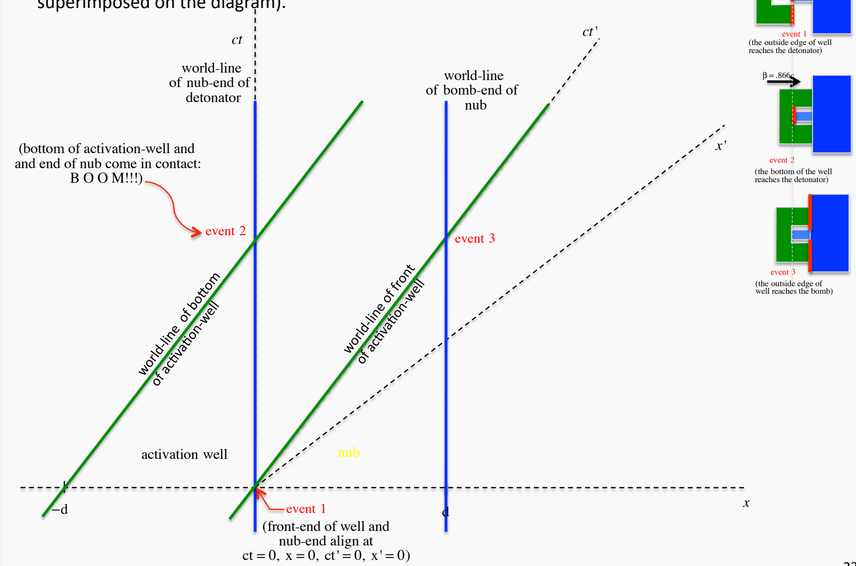
Putting together the world-lines of the *front-end* and *bottom of the activator-well*, as viewed from the bomb's frame of reference, we get:



Putting everything together: The space-time diagram below tracks the world-lines of the ends of the detonator-nub, assumed "stationary" in this frame, and the world-lines of the front-end and bottom of the activator-well, assumed "moving." On it, the nub at event 2 clearly fits into the well (that is, the activator's front-end strikes the bomb's front-end just as the nub strikes the bottom of the activation-well with the bomb going BOOM!).

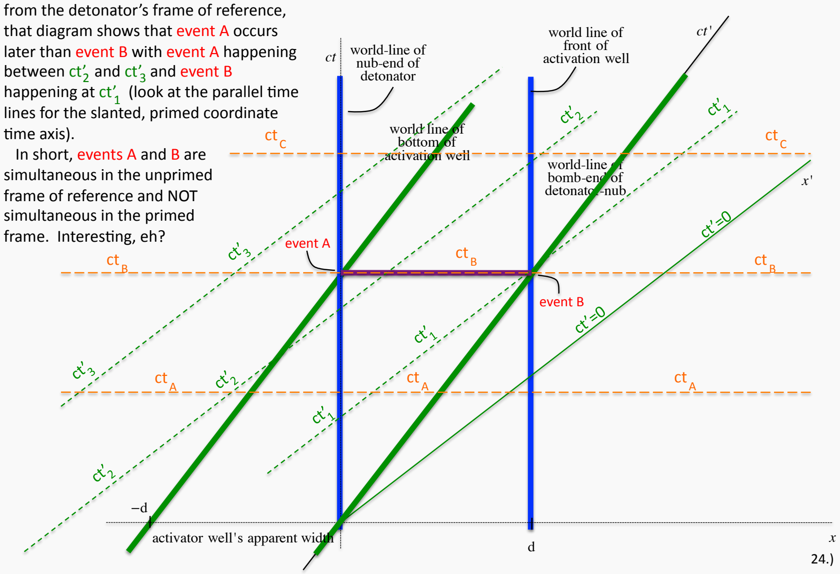


This is a more bare-bones version of the world lines and events in question. It is more like what you would see in a textbook or on a test (that is, pictures are not superimposed on the diagram).

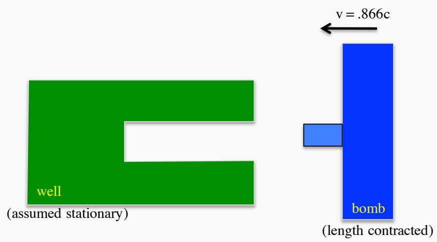


Parenthetical comment: Look at the diagram from the unprimed perspective. Both event 2 and event 3 occur at the same time in that frame (LOOK AT THE SKETCH—both events sit on the same horizontal time line at ct_B). That's what our sketch, drawn to scale, maintains has happened. What is interesting is that looking at those two events from the detonator's frame of reference, that diagram shows that event A occurs later than event B with event A happening between ct'_2 and ct'_3 and event B happening at ct'_1 (look at the parallel time lines for the slanted, primed coordinate time axis).

In short, events A and B are simultaneous in the unprimed frame of reference and NOT simultaneous in the primed frame. Interesting, eh?

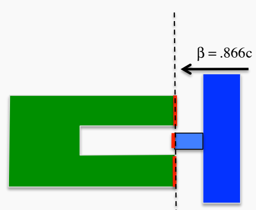


So far, so good. Let's now look at the system from the frame of reference of the activator well (that is, assume the activator well is stationary and the nub and bomb are moving to the left). The relativistic factors still hold at $\beta = .866$ and $\gamma = 2$, but now the activator-well (being assumed to be stationary) has a length of 2d, as originally defined, and the nub and bomb are length contracted. The nub is now $d/\gamma = d/2$ units in length. See sketch below.



Although it is now the bomb and nub that are length contracted, there are still three points of interest (events) to examine. They are identified below:

1.) event 1: The *outside edge of the activator-well* reaches the *nub-end of detonator*; this happens in the unprimed frame at x = 0, ct = 0. For the sake of completeness, this happens in the primed system at:

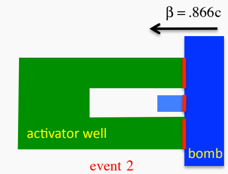


event 1
(the nub-end of detonator reaches the outside edge of activator)

$$x' = \gamma(x - \beta ct) = 2\left(0 - \left(\frac{\sqrt{3}}{2}\right)c(0)\right) \Rightarrow x' = 0$$

$$t' = \gamma(ct - \beta x) = 2\left(c(0) - \left(\frac{\sqrt{3}}{2}\right)(0)\right) \Rightarrow t' = 0$$

2.) event 2: The *bomb-end of the moving detonator-nub* aligns with the *outside-edge of the stationary activator* (it also comes in contact with the *front edge of the bomb*); in bomb's frame, this happens at x = d, ct = d/ β.



event 2
(the outside edge of well reaches the bomb)

Again, for completeness, in the activator's primed frame this happens at:

$$x' = \gamma(x - \beta ct) = 2\left(d - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{d}{\left(\frac{\sqrt{3}}{2}\right)}\right)\right) \Rightarrow x' = 0$$

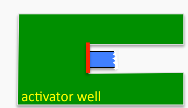
$$t' = \gamma(ct - \beta x) = 2\left(\left(\frac{d}{\left(\frac{\sqrt{3}}{2}\right)}\right) - \left(\frac{\sqrt{3}}{2}\right)(d)\right) = 2d\left(\left(\frac{1}{\left(\frac{\sqrt{3}}{2}\right)}\right) - \frac{\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)}\right)$$

$$= 2d\left(\left(\frac{1}{\left(\frac{\sqrt{3}}{2}\right)}\right) - \frac{\left(\frac{3}{4}\right)}{\left(\frac{\sqrt{3}}{2}\right)}\right) = 2d\left(\left(\frac{2 - \frac{3}{2}}{\sqrt{3}}\right)\right) = 2d\left(\frac{1}{2\sqrt{3}}\right)$$

$$\Rightarrow t' = \frac{d}{\sqrt{3}}$$

Note: In the activator's frame of reference, the *outside edge of the well* was originally defined (see slide 4) as being at x' = 0. In the activator's frame, that point hasn't changed position (the activator is stationary in its frame). In other words, the calculated x' value makes sense.

3.) event 3: The *nub-end of the detonator* reaches the *bottom of activator-well* and the bomb goes off; in bomb's frame, this happens at x = 0 at time ct = d/β .



event 3
(the nub of detonator reaches bottom of well . . . assuming it could)

Once again, for completeness, in the activator's frame this happens at:

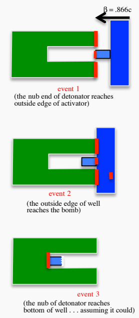
$$x' = \gamma(x - \beta ct) = 2\left(0 - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{d}{\left(\frac{\sqrt{3}}{2}\right)}\right)\right) \Rightarrow x' = -2d$$

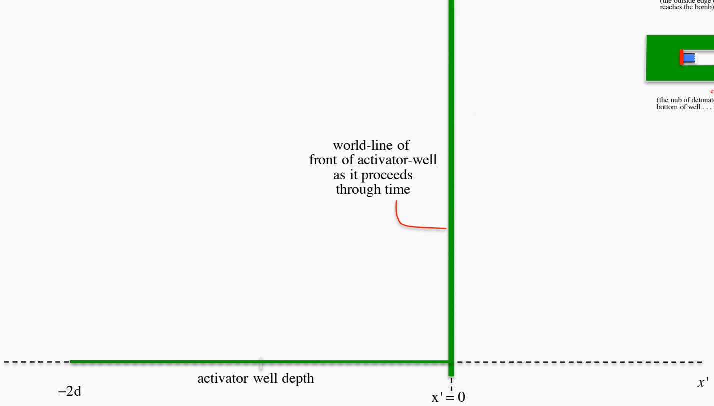
$$t' = \gamma(ct - \beta x) = 2\left(\left(\frac{d}{\left(\frac{\sqrt{3}}{2}\right)}\right) - \left(\frac{\sqrt{3}}{2}\right)(0)\right) \Rightarrow t' = \frac{4d}{\sqrt{3}}$$

Note: Again, this x' coordinate makes sense. The well in the activator's frame had a depth of 2d. As it was located to the left of x' = 0, it isn't surprising that its calculated value would be x' = -2d (hee, hee—isn't this fun?).

As before, we are going to do the space-time diagram in pieces.

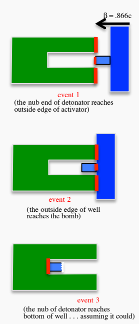
In the activator-well's frame of reference, assumed to be stationary, the primed coordinates are at right angles to one another. The front of the activator-well is defined to be at x' = 0, so its world-line proceeds through the origin along the ct' axis (remember, the world-line for a point that has a fixed position always runs parallel to the time axis). The world-line for the *front of the activator-well* is shown below.

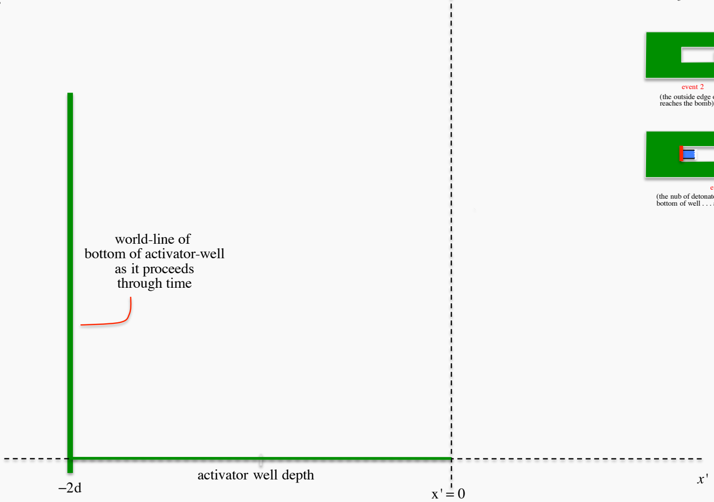




29.)

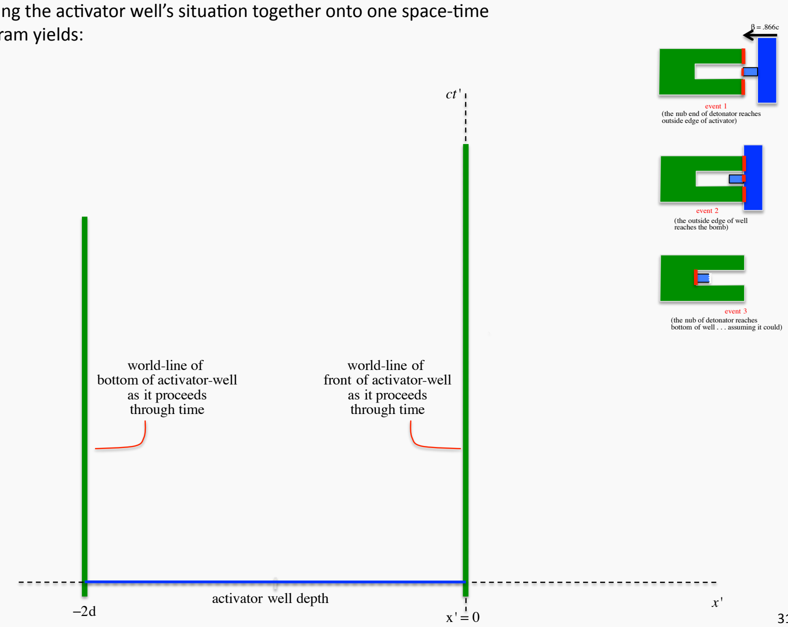
The activator well's depth in the activator's frame of reference was 2d. As the *bottom of the activator well* is to the left of x' = 0, its coordinate is always x' = - 2d. The world-line for the *bottom of the activator-well* is shown below.





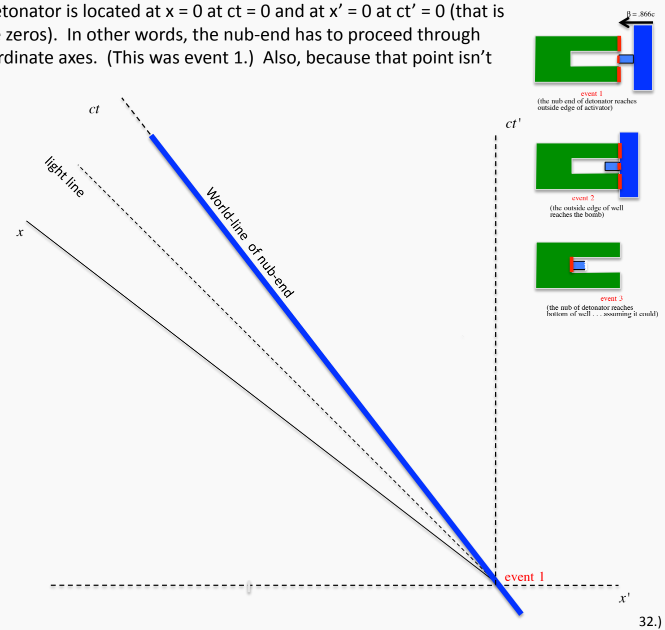
30.)

Putting the activator well's situation together onto one space-time diagram yields:



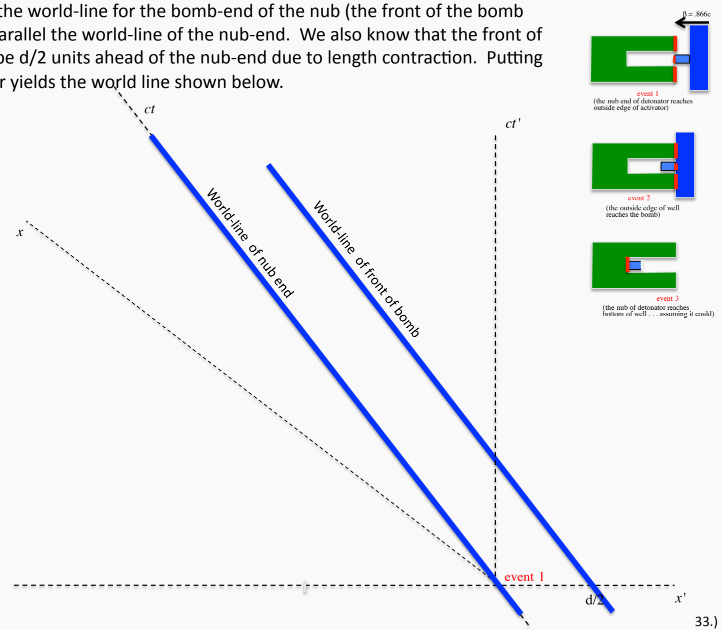
31.)

The nub-end of the detonator is located at x = 0 at ct = 0 and at x' = 0 at ct' = 0 (that is how we defined those zeros). In other words, the nub-end has to proceed through the origin of both coordinate axes. (This was event 1.) Also, because that point isn't moving in the unprimed frame, it's x coordinate must always be the same in that frame. Remembering that a point that is fixed in space has a world-line that is parallel to the time axis--in this case, the ct = 0 line--we get the world-line shown to the right.

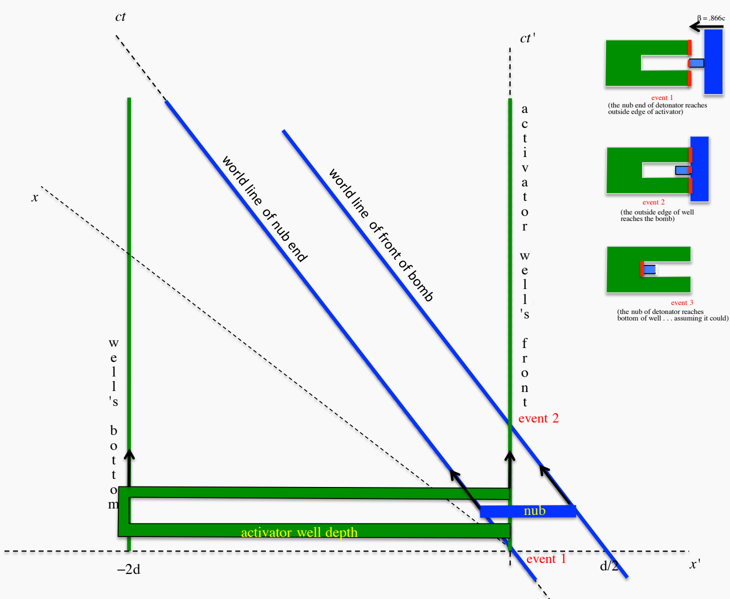


32.)

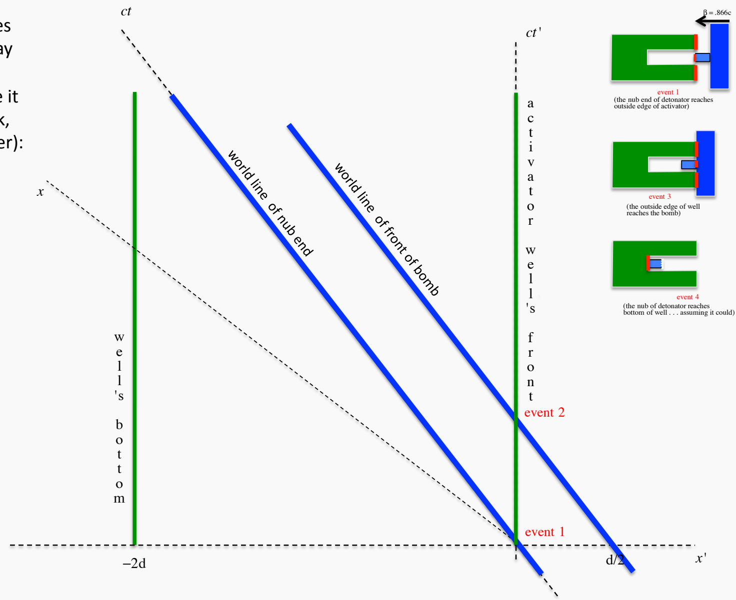
We know that the world-line for the bomb-end of the nub (the front of the bomb package) will parallel the world-line of the nub-end. We also know that the front of the bomb will be d/2 units ahead of the nub-end due to length contraction. Putting this all together yields the world line shown below.



Putting the four world-lines together with a little artwork (the artwork will be removed in the next slide), we get the following space-time diagram.



In bare-bones form (the way you would normally see it in a textbook, or where ever):

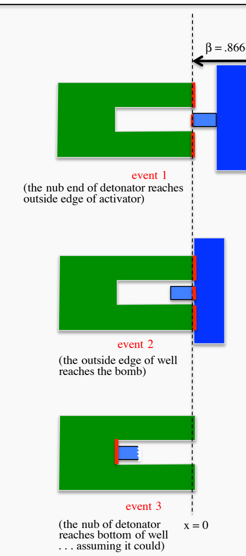


Here comes the fun stuff: It seems reasonable to assume that when the *front of the bomb* hits *the front of the activator-well* (event 2), the bomb and nub will crumple together the same way the hood of a stationary car and the hood of a speed car will crumple together if the speeding car rams into the stationary car. What's more, the *sketch* of our activator/nub situation makes it look as though those two crumpled surfaces will bring the activator to a stop long before event 3 can come to pass (that is, it looks as though the *nub-end* will never reach the *bottom of the well* so the bomb will never be activated).

I have only one word in response to that.

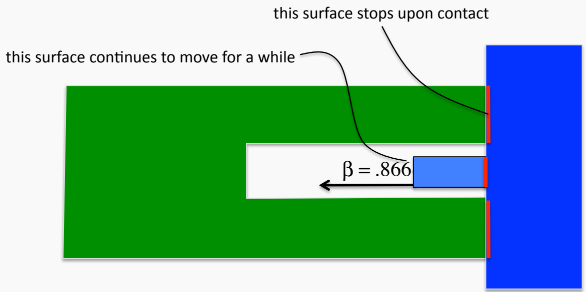
OH, CONTRARE! (well, OK, maybe two words)

But how can I make such an outrageous statement?



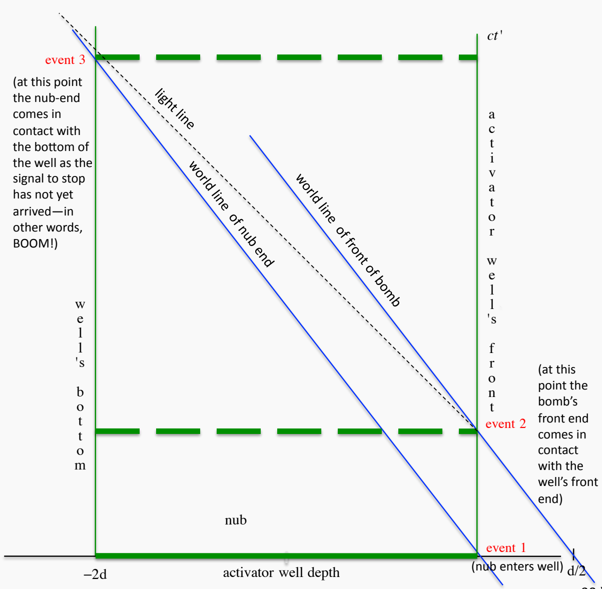
When event 2 occurs, the *bomb's front end* and the *activator's front surface* immediately begin to slow toward rest with respect to one another. That's all fine and well for those two surfaces, but how does the left-end of the nub located "d" units from the bomb's front end know it's supposed to stop? That is, what is the physical mechanism that brings the nub's end to a halt?

That mechanism is the consequence of a kind of domino effect. Molecules at the collision surface slow thereby pulling back on the molecules next to them. Those molecules slow pulling back on the molecules next to them. In other words, it is as though individual molecules are responding to a signal that propagates through the material, a signal whose speed is governed by the elastic nature of the molecular bonding of the structure.

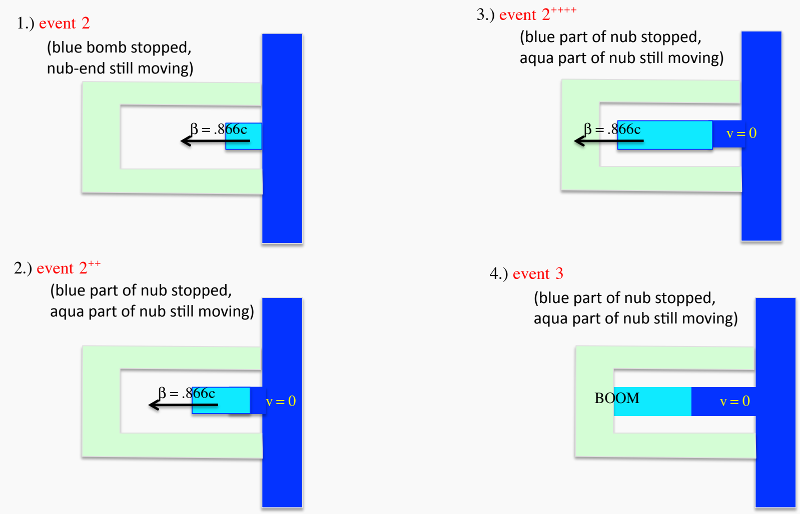


So let's assume this signal is emitted when the collision occurs. Let's also assume the signal moves at the speed of light (being a mechanical effect, the actual "signal" is nowhere close to that fast, but assume it is). Putting a light-line on our space-time diagram emanating from event 2, we see that the light will arrive at the *nub-end* (the left vertical line on the space-time diagram) AFTER event 3 occurs (i.e., after the *nub-end* strikes *the bottom of the activator-well*). In other words, the *nub-end* will set off the bomb BEFORE the light beam signal reaches it signaling that it is time to begin slowing.

As the actual signal moves much, much slower than light, this effect is even more pronounced and the *nub-end* will positively set off the bomb.



Put a little differently, during the time it takes the signal to reach the end of the nub, the nub will have apparently GROWN to the length of the well (plus some if the bomb doesn't go off) and its contact with the well's bottom will set off the bomb. Graphically (altering the sketch some to accommodate commentary), this looks like:



As I said before,

AIN'T THAT COOL!!!